1. (a) Let $\mathbf{a}, \mathbf{b},$ and $\mathbf{c}$ be non zero vectors in $\mathbb{R}^3$. Show that,
   i. $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = [\mathbf{b}, \mathbf{c}, \mathbf{a}] = [\mathbf{c}, \mathbf{a}, \mathbf{b}]$.
   ii. $[\mathbf{a}+\mathbf{b}, \mathbf{b}+\mathbf{c}, \mathbf{c}+\mathbf{a}] = 2[\mathbf{a}, \mathbf{b}, \mathbf{c}]$.

   (b) Prove that a necessary and sufficient condition for the points A, B, C, having position vectors $\mathbf{a}, \mathbf{b},$ and $\mathbf{c}$ relative to the origin $\mathbf{O}$, and $\mathbf{O}$ to be on a plane is that $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = 0$.

   (c) If the points A, B, and C, not all lying on the same straight line, have position vectors $\mathbf{a}, \mathbf{b},$ and $\mathbf{c}$ relative to a given origin, show that $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}$ is a vector perpendicular to the plane of A, B, and C.

2. (a) Evaluate $\int_C (x^2+y^2)dx + 2xydy$ for $C : x = \cos t, y = \sin t, 0 \le t \le \pi$.

   (b) Find constants a, b, c so that $\mathbf{F} = (x+2y+az)\underline{i} + (bx-3y-z)\underline{j} + (4x+cy+2z)\underline{k}$ is irrotational.

   (c) Find the scalar potential for $\mathbf{F}$.

3. (a) Verify Green's Theorem in the plane for

$$\oint_C (xy+y^2)dx + x^2dy$$

   where the curve C is the boundary of the region bounded by $y = x$ and $y = x^2$.

   (b) Use Divergence Theorem to evaluate the integral

$$\iint_S \left[(\frac{1}{2}x^2 + \exp(\cos zy))\underline{i} + (yx + \ln|z|)\underline{j} + (\tan xy)\underline{k} \right] \cdot d\overrightarrow{S}$$

   where S is a closed surface bounded by $y = 0, z = 0, z = 1-x^2,$ and $y = 2-z$.

4. (a) Let f be a real valued function such that

$$f(x,y) = \frac{x^2}{x^2-y^2}.$$

   Check whether the following statements are true or not (justify your answers)
   i. The domain of f is $\mathbb{R}^2 - \{(0,0)\}$.

ii. Limit of f at $(0, 0)$ does not exist.

(b) Use Stoke's Theorem to evaluate the line integral

$$\oint_C (z^2 - y^2)dx + (x^2 - z^2)dy + (y^2 - x^2)dz$$

where the curve C is formed by intersection of the paraboloid $z = 5 - x^2 - y^2$ with the plane $x + y + z = 1$.

The End.